On singularities of
spherically symmetric backgrounds
in string theory

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Abstract

We suggest that for singular rotationally invariant closed string backgrounds which need sources for their support at the origin (in particular, for special plane waves and fundamental strings) certain 'trivial' α'-corrections (which are usually ignored since in the absence of sources they can be eliminated by a field redefinition) may play an important role leading to the absence of singularities in the exact solutions. These corrections effectively regularize the source delta-function at the scale of $\sqrt{\alpha'}$. We demonstrate that similar smearing of the singularity of the leading-order point-charge solution indeed takes place in the open string theory.

September 1995

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1. Introduction

Fundamental string theory is intrinsically non-local with an effective space-time cutoff of order $\sqrt{\alpha'}$. The non-locality is reflected, e.g., in the presence of terms of all orders in α' in low-energy effective action. It suggests that short distance divergences should be absent not only in the quantum string loops but also in the classical solutions (both vacuum ones and the ones supported by sources). Singularities of leading-order solutions may disappear once α'-corrections are summed up. For example, given that the Schwarzschild solution gets non-trivial α'-modification at the next to the leading order [1], one may expect the $r = 0$ singularity may be absent in the exact metric (which may look like, e.g., as follows [2,3] $G_{00} = -1 + \frac{2M}{r} \exp(-c\frac{\alpha'M}{r^3})$, $M = GM_0$, $G \sim L_{Planck}^2 \sim \alpha'$). The possibility that higher order α'-corrections may eliminate the singularity of the spherically symmetric solutions is supported by analogy with the open string theory where the point-like singularity of the Coulomb solution is indeed 'smeared out' once the Maxwell action is replaced by the Born-Infeld plus derivative corrections one (see below).

One may expect that since the first-quantized fundamental string is effectively a non-local object, the background produced by it acting as a source should be non-singular. If one considers a closed string in a flat space with a large compact dimension y, the long-distance interactions of the two macroscopic winding string states ($y = \sigma$, $t = \tau$) can be described as a motion of one string in the 'massless' field of the other [4]. This corresponds to taking into account only leading-order interactions of the winding string with the massless string modes. The resulting weak-field solution to the Laplace-type equations can be generalized [5] to the solution of the leading-order effective string equations (this corresponds to taking into account interactions between 'soft' massless string modes). This macroscopic string or 'fundamental string' (FS) solution [5] is valid only in the weak-field region and thus its apparent singularity at the position of the string source ($r = 0$) should not be taken too seriously. Indeed, near $r = 0$ where the field strengths blow up the α'-corrections should become important and may eliminate the singularity.[1]

One possible approach is to define the fundamental string background directly at the string world-sheet level in terms of a conformal sigma model [11,12]. The resulting

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[1] The fundamental string solution [5] (in various variables, see, e.g., [6,7]) plays an important role in the arguments suggesting relations between different superstring theories in various dimensions [8,9,10]. Sometimes an emphasis is made on singularity of the FS solution when represented as a solution of the original theory and its non-singularity when expressed in terms of the variables of the dual theory (and interpreted as a soliton of the latter). It seems that the discussion of singularity or regularity of a solution of the leading-order (supergravity) equations may not have much sense since near the singularity higher order corrections should be taken into account. What is more relevant is whether a given solution needs or does not need a source for its support at the origin.

conformal invariance conditions ($R_{\mu\nu} + ... = 0$, etc.) are satisfied at $r = 0$ without need for a source. It turns out that there exists a scheme in which the leading-order FS solution is not modified by α'-corrections and thus remains singular. The singularity of the FS background may actually be absent at the level of the corresponding conformal field theory.

Alternatively, one may start with the leading-order string effective equations ($R^{\mu\nu} + ... = 0$, etc.) and interpret the apparent presence of the singularity at the origin as suggesting that the FS solution should be supported by a source. Here we shall follow this original approach of [4,5] and consider FS solution as corresponding to the combined 'field+source' action

$$\hat{S} = S(\varphi) + I(x, \varphi) = \int d^D x \sqrt{G} \ e^{-2\phi}[R + 4(\partial\phi)^2 - \frac{1}{12}H^2_{\mu\nu\lambda} + O(\alpha')] \qquad (1.1)$$

$$+ \frac{1}{4\pi\alpha'} \int d^2 z \sqrt{\gamma} \ [\partial_m x^\mu \partial^m x^\nu G_{\mu\nu}(x) + \epsilon^{mn} \partial_m x^\mu \partial_n x^\nu B_{\mu\nu}(x) + \ \alpha' R^{(2)} \phi(x)] \ .$$

The latter contains both the effective action for the background fields $\varphi = (G, B, \phi)$ (condensates of massless string modes) and the action of a source string interacting with the background. Though such mixture of actions may look strange from the point of view of the fundamental string theory, $\hat{S}$ can be interpreted as representing a non-perturbative 'thin handle' or 'wormhole' resummation of quantum string theory [13] (see also [14]). Extrema of $\hat{S}$ can be viewed as describing a semiclassical approximation (both in string coupling e^ϕ and in α') in which one 'big' macroscopic string is interacting with a condensate of massless string modes. The structure of the effective action S is not unambiguous: some coefficients in S change under local covariant field redefinitions which preserve the string S-matrix [15]. The important observation is that such redefinitions are no longer allowed in $\hat{S}$ since they transform also the second string source term. The solutions corresponding to $\hat{S}$ will thus be sensitive to an *off-shell* form of the effective action S which supplements I in the exponent after a resummation of string perturbate expansion [13]. In particular, they will depend on 'propagator corrections' to S (which are usually dropped out since they can be eliminated by a field redefinition).

As we shall discuss below, in the spherically symmetric cases where the singularity at the origin can be attributed to the presence of a source one cannot a priori ignore even these 'redefinable' α' corrections. Under a certain natural assumption about the structure of these corrections their effect will be to completely smooth out the singularity. Since (in the presence of source-related singularities) different off-shell extensions of S may lead to different results one should not thus disregard the possibility that the singularity is avoided in the exact solution. It remains to be understood if there is an additional 'bootstrap' condition that may fix the structure of the action/solutions in a unique way.

The issue of the singularity of the FS solution will be addressed in Section 2. We shall first discuss the spherically symmetric plane-wave solution which is related to the FS background by a duality transformation.

In Section 3 we shall demonstrate that the spherically symmetric solution corresponding to a point-like source in the open string theory is non-singular, with the singularity of the leading-order solution being eliminated by α'-corrections.

Section 4 will contain some concluding remarks.

2. Singular spherically symmetric solutions in closed string theory

The solutions we are going to discuss below are special (and simpler than, e.g., Schwarzschild) in that, ignoring the presence of sources, one could formally argue that there exists a special scheme in which they are not modified by α'-corrections [11,12]. Assuming that one starts from the effective action and considers $G_{\mu\nu}$ as a fundamental variable, i.e. $R^{\mu\nu} + ... = 0$ as the fundamental equations, these solutions need to be supported by sources. As a result, there is no longer an equivalence between different off-shell extensions and there is a possibility that α'-corrections can lead to a 'regularization' the delta-function source thus eliminating the corresponding singularity.

2.1. Plane wave solution

We shall first discuss the plane-wave type solution described by the following sigma model $(\mathcal{R} = \frac{1}{4}R^{(2)})$

$$L = \partial u \bar{\partial} v + K(x)\partial u \bar{\partial} u + \partial x^i \bar{\partial} x_i + \alpha' \mathcal{R} \phi_0 . \tag{2.1}$$

This model is formally conformal to all orders in α' provided [16]

$$\Delta K = 0 , \qquad \Delta \equiv \partial^i \partial_i . \tag{2.2}$$

There are no α'-corrections assuming one uses the minimal subtraction scheme in deriving the β-functions, or equivalently, ignores the propagator corrections in the effective action. The condition (2.1) has the standard plane-wave solution $K = a_i x^i + m_{ij} x^i x^j$, $m_{ii} = 0$. It admits (for $r > 0$) also the rotationally symmetric solution [17] ($K = K(r)$, $r^2 = x^i x_i$, $i = 1, ..., N = D - 2$)

$$K = 1 + \frac{M}{r^{N-2}} , \quad N > 2 , \tag{2.3}$$

$$K = -M \ln \frac{r}{r_0} , \quad N = 2 ,$$

which is singular at the origin where the components of the curvature blow up. The conformal invariance equation (2.1) is *not* actually satisfied at the origin: the trace of the 2d stress-energy tensor is $T^m_m \sim \delta^{(N)}(x)\partial u \bar{\partial} u$, i.e. it receives a non-trivial contribution which is local in space-time but 'global' on the world sheet. Thus the The model is *not* conformal unless some extra assumptions are made, e.g., the line $r = 0$ is 'cut' out off the space or an external δ-function source is added.

Indeed, (2.2) is a solution of the Poisson equation ($\mu = M(N - 2)$ vol S^{N-1})

$$\Delta K = -\mu \delta^{(N)}(x) . \tag{2.4}$$

One may try to interpret the δ-function term as corresponding to a classical string source.[2] Accepting the presence of a source one is to reconsider the issue of the α'-corrections to the l.h.s. of (2.4). In general, given a background (2.1) admitting a covariantly constant null Killing vector one still has a freedom of the following field redefinitions (note that all non-trivial second-rank tensors involving contractions of curvature tensors vanish on this background)

$$G_{\mu\nu} \to G_{\mu\nu} + b_1 \alpha' R_{\mu\nu} + b_2 \alpha'^2 D^2 R_{\mu\nu} + ... + b_n \alpha'^{n+1} D^{2n} R_{\mu\nu} + ... \tag{2.5}$$

This implies the replacement of the $R_{\mu\nu} = 0$ equation (2.2) by

$$R_{\mu\nu} + d_1 \alpha' D^2 R_{\mu\nu} + ... + d_n \alpha'^n D^{2n} R_{\mu\nu} + ... \equiv f(\alpha' D^2) R_{\mu\nu} = 0 , \tag{2.6}$$

which has the non-vanishing uu-component being equivalent to

$$f(\alpha' \Delta) \Delta K = 0 , \qquad f(z) = 1 + d_1 z + d_2 z^2 + ... \tag{2.7}$$

Eq. (2.7) is the analog of (2.2) in a generic scheme. The same function f appears also in the kinetic term in S. The simplest and most natural 'non-minimal' scheme choice corresponds to $f(z) = e^{-cz}$, $c = -d_1$, i.e.,

$$e^{-c\alpha' \Delta} \Delta K = 0 . \tag{2.8}$$

Indeed, to find the condition of conformal invariance of (2.1) one first integrates over u, v, obtaining the effective scalar ('tachyonic') vertex $K(x) \partial u_0 \bar{\partial} u_0$ (u_0 is the background value of u). The condition of the zero anomalous dimension of this interaction term then leads to (2.2). This is, however, a sufficient but not necessary condition for conformal invariance. The divergent term in the 2d effective action is $\exp(-\alpha' \log \epsilon \, \Delta) K(x) \partial u_0 \bar{\partial} u_0$ (ϵ is a 2d UV cutoff). Taking the derivative over $\log \epsilon$ and setting $\log \epsilon = c$ ($c \neq 0$ corresponds to a non-minimal subtraction scheme in which higher-order tadpoles contribute to the β-function) leads to (2.8).

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[2] In [17] the $D = 5$ solution (2.3) was interpreted as a field of an infinite string boosted to the speed of light. However, it does not seem to be possible to make such identification precise: the configuration $u = \sigma$, $v = \tau$, $x^i = 0$ produces upon the substitution into the equations which follow from (1.1) also a source $\sim \delta^{(N)}(x) \int d\sigma d\tau \delta(u) \delta(v) \partial_m u \partial^m u$ for the R^{uu}-equation which should be satisfied trivially in the case of (2.1). No other string configuration seems to work either (this applies also to more general models of [12] containing $K \partial u \bar{\partial} u$ term with K given by (2.3)). We shall ignore this complication since it is absent in the case of the FS solution we are mostly interested in. The latter can indeed be supported by a string source in a consistent manner [5].

One usually expects that physical properties of perturbative vacuum string solutions should not depend on a choice of parameters which represent the freedom of off-shell continuation. Indeed, the perturbative solutions of (2.7) or (2.8) are the same as of (2.2). This, however, is no longer true in the presence of *sources*. In particular, these equations are *not* equivalent in the case of the singular leading-order solution (2.3) which needs a δ-function source for its support. In what follows we shall use the simple exponential choice for the function $f(z)$ (2.8),

$$e^{-c\alpha'\Delta}\Delta K = -\mu\delta^{(N)}(x) \ . \tag{2.9}$$

Rewriting (2.9) as

$$\Delta K = -\mu\delta_{\alpha'}^{(N)}(x) \ , \tag{2.10}$$

$$\delta_{\alpha'}^{(N)}(x) \equiv e^{c\alpha'\Delta}\delta^{(N)}(x)$$

$$= \int \frac{d^N k}{(2\pi)^N} \ \exp(ikx - c\alpha'k^2) = (4\pi c\alpha')^{-N/2} \exp(-\frac{x^2}{4c\alpha'}),$$

one can also interpret the replacement of (2.4) by (2.10) as as a result of 'smearing' of the δ-function source at the string scale $\sqrt{\alpha'}$.[3] This is analogous to the 'regularization' (at the scale of the inverse Higgs mass) of a similar δ-function source in the case of the Abrikosov-Nielsen-Olesen string (see, in particular, [18,19]). There the quantum effects transform the δ-function which describes the coupling of the 'axion' $B_{\mu\nu}$ (which is dual to angular part of the Higgs field) to the string into a non-singular function, e.g, $(2\pi)^{-4}\int d^4 k \exp(ikx - c\Lambda^{-2}k^2)$ [19]. Note also that the transformation (2.5) or $K \to f(\alpha'D^2)K$ which relates (2.2) and (2.9) can be interpreted as a modification of the field–string coupling in the string source action term in (1.1).

Assuming $c > 0$ the solution of (2.10) is *regular* at $r = 0$ and reduces to (2.3) at large r. For example, in the case of $D = 5$, i.e. $N = 3$ it is given by (cf. (2.3))

$$K = 1 + \frac{M}{r}\mathrm{erf}\Big(\frac{r}{2\sqrt{c\alpha'}}\Big) \ , \qquad \mathrm{erf}(b) = \frac{2}{\sqrt{\pi}}\int_0^b dz \ e^{-z^2} \ . \tag{2.11}$$

The conclusion about regularity then applies also to the $a = \sqrt{3}$ 'Kaluza-Klein' extreme electric black hole solution which is obtained from the $D = 5$ solution (2.1) by dimensional reduction [17] ($\mathcal{A}_\mu$ is the vector field and σ is the modulus)

$$ds_4^2 = -K^{-1}(r)dt^2 + dr^2 + r^2 d\Omega_2^2 \ , \quad \mathcal{A}_t = K^{-1}(r) \ , \quad \sigma = \frac{1}{2}\ln K(r) \ . \tag{2.12}$$

To summarize, starting with a generic choice of the effective action and studying how the leading-order rotationally-symmetric solution (2.3) is modified by α'-corrections one learns that because of the necessity to introduce a source at the origin the α'-corrections cannot be ignored and may completely eliminate the singularity.

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[3] This smearing can be attributed to the quantum fluctuations of the first-quantized string source which make the interaction of the string with the background fields non-local. Indeed, taking the expectation value of the δ-function $\delta(x(\sigma,\tau) - x)$ corresponds to replacing e^{ikx} by $e^{ikx-c\alpha'k^2}$, $c = \log \epsilon$ in its Fourier representation.

2.2. Fundamental string solution

The FS solution [5] is described by the following 2d sigma model action [11]

$$L = F(x)\partial u \bar{\partial} v + \partial x^i \bar{\partial} x_i + \alpha' \mathcal{R}\phi(x) , \qquad \phi = \phi_0 + \frac{1}{2}\ln F(x) . \tag{2.13}$$

This action is related to (2.1) by the duality transformation (if we set $u = y - t$, $v = y + t$ then the duality $y \to \tilde{y}$ leads to (2.1) with $u = \tilde{y}$, $v = t$, $K = F^{-1}$). Expanding near a 2d background (u_0, v_0, x_0) one finds that the condition of conformal invariance of (2.13) is

$$\Delta F + 2F^{-1}\partial^i F \partial_i F = F^2 \Delta F^{-1} = 0 , \tag{2.14}$$

which is equivalent to $R_{\mu\nu} + ... = 0$. As was argued in [11], there exists a scheme in which (2.14) represents the exact conformal invariance condition to all orders in α'.

If, instead, one introduces 2d sources J_u, J_v for u, v and then integrates u, v out, one ends up with the effective 'tachyonic' vertex $F^{-1}(x)J_u\bar{J}_v$ so that the condition of conformal invariance is just

$$\Delta F^{-1} = 0 , \tag{2.15}$$

as in the dual model (2.1) with $K = F^{-1}$. The condition (2.15) follows from $R^{\mu\nu} + ... = 0$ (which is the same equation that is obtained from the effective action by varying $G_{\mu\nu}$). It is equivalent to (2.14) provided F is non-vanishing everywhere. This is no longer true in the case of the singular FS solution [5] with $F^{-1} = K$ given by (2.3). Since F does vanish (for $D > 4$) at $r = 0$ eqs. (2.14) and (2.15) are not equivalent: while (2.14) holds everywhere, (2.15) needs to be supported by a source at the origin.

Following [5] let us assume that the source representing a macroscopic string state should indeed be added to the r.h.s. of (2.15). The FS background is then a consistent leading-order solution of the equations corresponding to the action (1.1) supported by the source produced by the string configuration $u = \sigma + \tau$, $v = \sigma - \tau$, $x^i = 0$ (which remains a solution of the string equation also in the case of the non-trivial background (2.13)). As in the plane-wave case discussed in the previous subsection, we are then to address the issue of α'-corrections to the leading terms in the effective action, i.e. to the l.h.s. of eq. (2.15). As discussed above, the intrinsic non-locality of the 1-st quantized string suggests that in the presence of a source one should replace the Δ-operator in (2.15) by a 'regularised' expression $f(\alpha'\Delta)\Delta$ as in (2.7). For the simplest exponential choice of f the resulting equation is then equivalent to (2.9) and thus the $r = 0$ singularity of the FS solution is smeared out. Let us stress again that in the presence of a source it is no longer true that different choices of the function f or even of the constant c in (2.8) lead to physically equivalent solutions.

Similar conclusion should apply also to the generalisations [20,21,22,23] of the FS model (2.13), in particular, to the following one ($n = \pm 1$) [12][4]

$$L = F(x)\partial u \bar{\partial} v + n \partial x^i \bar{\partial} x_i + \alpha' \mathcal{R}\phi(x) , \qquad \phi = \phi_0 + \frac{1}{2}\ln F(x) . \qquad (2.16)$$

Such $D = 5$, $n = 1$ model describes (upon dimensional reduction to $D = 4$) the $a = 1$ extreme electric black hole background

$$ds_4^2 = -F^2(r)dt^2 + dx_i dx^i , \qquad \mathcal{A}_t = -\mathcal{B}_t = F(r) , \qquad e^{2\phi} = F(r) . \qquad (2.17)$$

It is thus plausible that when the α'-corrections are included, this background is no longer singular at $r = 0$.

3. Regularity of the point-charge solution in the open string theory

In the examples of solutions discussed above the α'-corrections which were responsible for smearing of the singularity where of 'propagator' or 'redefinable' nature. In general, there are also non-trivial α'-corrections that cannot be eliminated by a field redefinition even in the absence of sources. The example of the open string theory discussed below suggests that their effect is also to smooth out the singularity of a leading-order solution.

Namely, we shall show that the point-charge singularity of the Maxwell theory is absent in the open string theory. The tree level (disc) term in the abelian vector field effective action of the open (super)string theory has the following structure [26,27,28]

$$S = e^{-\phi_0} \int d^D x \left[\sqrt{-\det\left(\eta_{\mu\nu} + 2\pi\alpha'\mathcal{F}_{\mu\nu}\right)} + \alpha'^3 f_1(\alpha'\mathcal{F})\partial\mathcal{F}\partial\mathcal{F} + ... \right] , \qquad (3.1)$$

where $f_1 = a_1\alpha'^2\mathcal{F}^2 + ...$ Let us first ignore all field strength derivative $O(\partial\mathcal{F})$ terms and consider just the first Born-Infeld term. Adding a charged open string source to (3.1) (which can be represented by a point-particle source term $\sim QA_0(x)\delta^{(D-1)}(x)$ since the open string is charged only at its ends) one may find the corresponding electric field.

A remarkable property of the Born-Infeld Lagrangian is that while in the Maxwell theory the field of a point-like charge is singular at the origin and has infinite energy, in the Born-Infeld case the field is regular at $r = 0$ (where the electric field

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[4] Let us note also that all charged fundamental string solutions discussed in [21] and the first reference in [8] are described by the special chiral null model [12] (y^I are internal toroidal coordinates) $L = F(x)\partial u[\bar{\partial} v + K(x)\bar{\partial} u + 2A_u^I(x)\bar{\partial} y^I] + L_{IJ}\partial y^I \bar{\partial} y^J + \partial x^i \bar{\partial} x^i + \alpha' \mathcal{R}\phi(x)$, or its straightforward 'heterotic' generalisation in which part of y^I are represented by chiral scalars (see [24,25]).

takes its maximal value) and its total energy is finite [29]. For example, in $D = 4$, $\sqrt{-\det(\eta_{\mu\nu} + 2\pi\alpha'\mathcal{F}_{\mu\nu})} = \sqrt{1 - (2\pi\alpha'E)^2}$ so that the analogue of the Maxwell equation is $\partial_r(r^2\mathcal{D}) \sim Q\delta(r), \quad \mathcal{D} \equiv E/\sqrt{1 - (2\pi\alpha'E)^2}$. The solution is $\mathcal{D} = Q/r^2$, i.e.,

$$E \equiv \mathcal{F}_{rt} = \frac{Q}{\sqrt{r^4 + r_0^4}} , \qquad r_0^2 \equiv 2\pi\alpha'Q . \tag{3.2}$$

From the point of view of the distribution of the electric field ($\rho = \text{div } E/4\pi$) the source is no longer point-like but has an effective radius $r_0 \sim \sqrt{\alpha'Q}$.

The electric field is approximately constant ($E \sim Q/r_0^2 \sim 1/\alpha'$) in the region $0 \leq r < r_0$. Its derivative vanishes at $r = 0$ and is suppressed by a power of Q near $r \sim r_0$ ($\partial_r E \sim Q/r_0^3 \sim \alpha'^{-3/2}Q^{-1}$). That means that the effect of the derivative terms in (3.1) should be small, i.e. the qualitative conclusion about the regularity of the static spherically symmetric point source solution applies to the *full effective action* of the open string theory.[5]

4. Concluding remarks

We have argued that in the cases when there are no 'non-trivial' α'-corrections but there is a source at the origin (so that the string equation reduces, e.g., to the Poisson equation with a δ-function source) one may not be able to ignore the effect of the 'trivial' α'-corrections. Then the exact solution may turn out to be non-singular. The 'smoothening' of the singularity of the solution corresponding to a fundamental string source seems to be natural: the quantum string has a built-in cutoff at the space-time scale $\sqrt{\alpha'}$ and like the Abrikosov-Nielesen-Olesen string (for a finite Higgs mass) should be non-singular.

The suggested absence of singularities in the solutions produced by local string sources is supported by the example of the open string theory where the α'-corrections eliminate the singularity of the Coulomb solution. This strengthens the expectation that the exact version of the Schwarzschild solution should also be non-singular.

There are many open questions remaining. One is the existence of a well-defined CFT description behind such solutions with sources. Another is about a formulation of conformal invariance conditions in the presence of sources which effectively chooses the 'right' form of the off-shell effective action (or, as in the absence of sources, is not sensitive to field redefinitions). Given that (1.1) corresponds to a resummation of the standard string perturbation theory [13] such formulation should be related to the attempts to extend the condition of conformal invariance to the string loop level [30,31]. It should be useful also to understand the reason behind the strange feature of the fundamental string

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[5] Moreover, if Q is large the derivative corrections to the effective action do not significantly modify the form of the Born-Infeld solution (3.2).

background that it needs or does not need a source for its support depending on the form of the Einstein equations (with raised or lowered indices) one chooses to solve and depending on the metric one uses (string metric or the one rescaled by a function of the dilaton).[6]

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[6] The change of variables (conformal rescaling of the metric) used, e.g., in [6,32] seems to eliminate or introduce a source in the leading-order string equations. The Einstein equation with raised indices and a source in the r.h.s. $R^{\mu\nu} + ... \sim \int d\sigma d\tau \delta(x - x(\sigma, \tau)) \partial x^\mu \bar{\partial} x^\nu$ is not invariant under the conformal rescalings of the metric. Rescaling the metric by a factor which vanishes at the origin one may effectively drop out the source contribution.

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